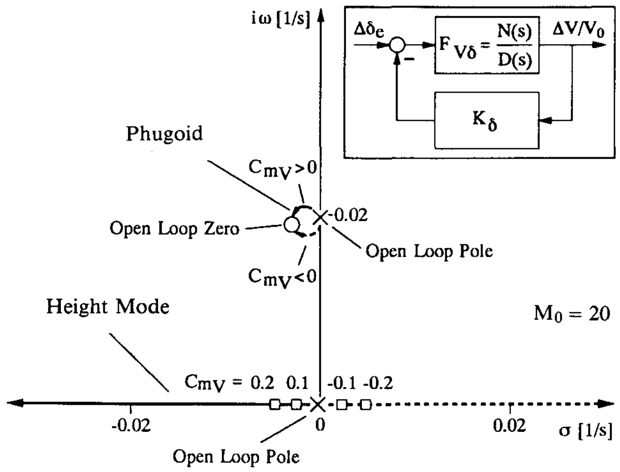

Fig. 3 Indirect thrust/speed effects (C_{m_V}) on phugoid and height mode, $M_0 = 20$.

cally) linear dependence of s_h on C_{m_V} for the range considered. This is in agreement with the approximation described in Eq. (6).

The effect of C_{m_V} is reduced for both long-term modes of motion at higher Mach numbers (Fig. 3). From this figure it follows that the indirect thrust/speed effects considered here show a reduction that is similar to the direct case (n_V) described in Fig. 1.

Conclusions

It is shown that direct thrust/speed effects have practically no influence on phugoid characteristics in hypersonic flight. The height mode is rather sensitive to this effect for lower hypersonic Mach numbers. At high hypersonic Mach numbers, the direct thrust/speed effect on the height mode is also reduced. By contrast, direct thrust/altitude effects are significant on both the phugoid and height mode. This holds throughout the hypersonic Mach number range.

Indirect thrust/speed effects are concerned with an influence on pitching moment due to thrust-axis offset. In hypersonic flight, this type of thrust/speed effect is significantly reduced as regards the phugoid. In particular, it may not cause aperiodic phugoid instability that is usually considered to be introduced when this effect is beyond a critical value. Rather, the phugoid exists as an oscillation, the stability of which may even be increased. However, the height mode is sensitive to indirect thrust/speed effects. For this mode of motion, aperiodic instability may be caused by negative values of pitching moment variation with speed due to thrust effects. For positive values, height mode stability is increased. At high hypersonic Mach numbers, indirect thrust/speed effects on the height mode are reduced.

In summary, direct and indirect thrust/speed effects are reduced in hypersonic flight. For the phugoid, this is valid throughout the whole hypersonic flight regime. There is also a reduced sensitivity of the height mode. This reduction in thrust/speed effects is valid for high hypersonic Mach numbers.

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Frequency Analysis of the Hoop-Column Antenna Using a Simplified Model

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Introduction

BECAUSE of its light weight, the hoop-column antenna is a candidate for future space flights of large flexible structures. During the past decade, a great deal of research activity, both theoretical and experimental, was directed toward developing a lightweight space antenna for future large flexible structures. The hoop-column antenna and the wrap-rib antenna are the two antennas being investigated at the present time. The hoop-column antenna is described in Refs. 1 and 2 (see Fig. 1). In this Note, we develop a simple model of the hoop-column antenna and compare the frequencies generated by the simple model with that of a complex finite element model.[3] Our frequencies compare favorably with those predicted by the more complex finite element model.[3]

Physical Model of the Hoop-Column Antenna

The hoop-column antenna basically consists of a shallow parabolic reflector made of membrane-like material supported by concentric hoops. The center of the reflector is fixed by a column (feed mast) that also supports the reflector at various points by a system of upper and lower cables (see Fig. 1). In this Note, we present a simplified model of the hoop-column antenna. The frequencies obtained from the model are compared with those of the more complex NASTRAN finite element model.[3]

The simplified model (see Figs. 2) of the hoop-column antenna consists of a circular membrane reflector and is based on the following assumptions:

1) The central annular radius is fixed by the column.

2) The column is rigid.

3) The support cables at the top and the bottom of the antenna (reflector) are modeled as two sets of massless identical springs arranged so that they are 120 deg apart around the circumference of the membrane (see Figs. 2).

This model differs from the classical problem of the vibration of a circular membrane in the following respects:

1) The present model is annular.

2) In the present model, the outer boundary (hoop) of the annular membrane is not fixed but moves subject to the constraints of the springs and the inner boundary is fixed by the column.

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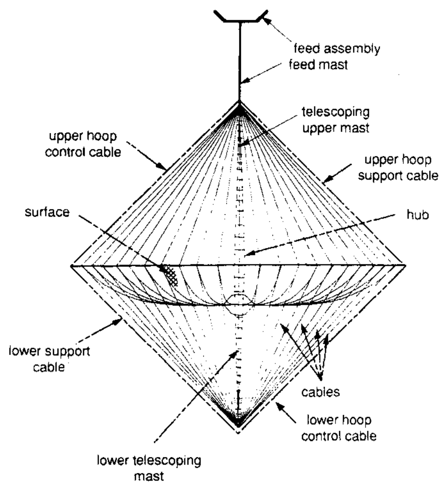

Fig. 1 Hoop-column antenna configuration.

Eigenfrequencies for the simplified model are obtained by solving the equations of motion (partial differential equations with appropriate boundary conditions) for the simple model of the hoop-column antenna.

Equations of Motion

A 100-m-diam point design hoop-column antenna is modeled as a flexible circular membrane with multiple boundary conditions. The outer radius moves with the hoop and has simple harmonic motion due to motion of the cables modeled as massless springs. The inner radius is fixed by the column. For symmetrical deflection, the deflection $W(r,t)$ is governed by the following partial differential equation:

$$\frac{\partial^2 W(r,t)}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial W(r,t)}{\partial r} = \left(\frac{1}{\alpha^2}\right)\frac{\partial^2 W(r,t)}{\partial t^2} \tag{1a}$$

and the following boundary and initial conditions:

$$W(e,t) = 0 \tag{1b}$$

$$W(a,t) = X(t) = A \sin \omega t \tag{1c}$$

$$W(r,0) = 0 \tag{1d}$$

$$\left.\frac{\partial W(r,t)}{\partial t}\right|_{t=0} = 0 \tag{1e}$$

where W is the deflection, ρ is the density of the membrane, T the tension, t the time, r the radial distance measured from the center, ω the vibrational frequency of the springs, A the amplitude of vibration, $X(t)$ the displacement of the springs due to simple harmonic motion, a the outer radius, e the inner radius, and $\alpha^2 = T/\rho$.

Solutions to the Equation of Motion

The general solution of the deflection $W(r,t)$ governed by Eqs. (1) can be expressed in the form

$$W(r,t) = R(r)T(t) \tag{2a}$$

$$T(t) = A_1 \cos(\alpha\lambda t) + A_2 \sin(\alpha\lambda t) \tag{2b}$$

$$R(r) = C_1 J_0(\lambda\, r) + C_2 Y_0(\lambda r) \tag{2c}$$

where A_1, A_2, C_1, and C_2 are constants of integration to be determined from boundary conditions. J_0 and Y_0 are zero-order Bessel functions of the first and second kind, respectively.

From Eqs. (2a–2c) and boundary conditions (1b) and (1c), it follows that

$$W(r,t) = A_2 C_2 \left[\frac{-Y_0(\lambda e) J_0(\lambda r)}{J_0(\lambda e)} + Y_0(\lambda r)\right] \sin \omega t \tag{2d}$$

$$A_2 C_2 = A / [Y_0(\lambda a) - Y_0(\lambda e) J_0(\lambda a)/J_0(\lambda e)] \tag{2e}$$

From Eqs. (2d) and (2e), it follows that the particular solution can be expressed in the form

$$W_p(r,t) = A\left[\frac{J_0(\omega e/\alpha) Y_0(\omega r/\alpha) - Y_0(\omega e/\alpha) J_0(\omega r/\alpha)}{J_0(\omega e/\alpha) Y_0(\omega a/\alpha) - Y_0(\omega e/\alpha) J_0(\omega a/\alpha)}\right] \sin \omega t$$

provided

$$J_0(\omega e/\alpha) Y_0(\omega a/\alpha) - Y_0(\omega e/\alpha) J_0(\omega a/\alpha) \neq 0$$

and the general solution can be expressed in the form

$$W(r,t) = [C_1' J_0(\lambda r) + C_2' Y_0(\lambda r)][A_1' \cos(\alpha\lambda t) +$$
$$A_2' \cos(\alpha\lambda t)] + W_p(r,t)$$

where C_1', C_2', A_1', and A_2' are constants to be determined from initial and boundary conditions (1b–1e). It follows from Eqs. (1b–1e) that

$$C_1' J_0(\lambda e) + C_2' Y_0(\lambda e) = 0$$

$$C_1' J_0(\lambda a) + C_2' Y_0(\lambda a) = 0$$

$$A_1' = 0$$

For nontrivial solutions for the constants C_1', C_2', the following determinantal equation must be satisfied:

$$\begin{vmatrix} J_0(\lambda e) & J_0(\lambda e) \\ J_0(\lambda a) & J_0(\lambda a) \end{vmatrix} = 0 \tag{3a}$$

Equation (3a) has an infinite number of roots, which we denote by λ_n, $n = 1,2,\ldots$. They are the eigenfrequencies of the system. The general solution for $W(r,t)$ can be expressed in the form

$$W(r,t) = A\left[\frac{J_0(\omega e/\alpha) Y_0(\omega r/\alpha) + Y_0(\omega e/\alpha) J_0(\omega r/\alpha)}{J_0(\omega e/\alpha) Y_0(\omega a/\alpha) - Y_0(\omega e/\alpha) J_0(\omega a/\alpha)}\right] \sin \omega t$$

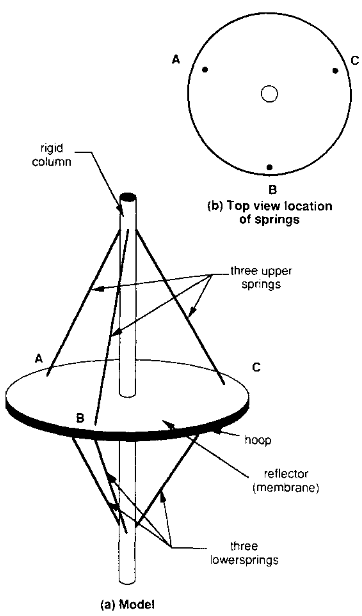

Fig. 2 Simple model of the hoop-column antenna: a) model, b) top view location of springs.

Table 1 Eigenfrequencies and amplitudes of a 100-m-diam hoop-column antenna ($a = 100$ m, $e = 1$ m, $\alpha = 1$, $\omega = 1$)

n	λ_n	A_n	λ_n (predicted by by the simple model[3])
1	0.244	$0.3200\ \Pi^2 A$	0.119
2	0.398	$0.4600\ \Pi^2 A$	0.214
3	0.513	$0.0016\ \Pi^2 A$	0.271
4	0.603	$0.0020\ \Pi^2 A$	0.506
5	0.681	—	0.721
6	0.754	—	0.890
7	0.823	—	0.919
8	0.956	—	1.009

Table 2 Comparison between the simple model and the finite element model

Antenna model	Ratio of eigenfrequencies						
Simple model	1.7	1.29	1.18	1.13	1.11	1.09	1.16
Finite element model[3]	1.8	1.26	1.87	1.42	1.13	1.03	1.18

$$+ \sum_{n=1}^{\infty} A_n \left[\frac{-Y_0(\lambda_n e) J_0(\lambda_n r)}{J_0(\omega e/\alpha) Y_0(\omega\ r/\alpha)} + Y_0(\lambda_n r) \right] \sin(\alpha \lambda_n t) \tag{3b}$$

The coefficients A_n, $n = 1,2,\ldots$, are to be obtained from the initial conditions (1b-1e) and the Bessel-Fourier series expansion and are given by[4]

$$A_n = \frac{\Pi^2 \lambda_n J_0(\lambda_n e)[L_1 - L_2 - L_3 + L_4]}{2\alpha[J_0^2(\lambda_n e)/J_0^2(\lambda_n a) - 1]} \tag{4a}$$

where

$$L_1 = \int_e^a r\ \bar{k}_1 Y_0\left(\frac{\omega r}{\alpha}\right) Y_0(\lambda_n e) J_0(\lambda_n r)\ \mathrm{d}r \tag{4b}$$

$$L_2 = \int_e^a r\ \bar{k}_1 Y_0\left(\frac{\omega r}{\alpha}\right) J_0(\lambda_n e) Y_0(\lambda_n r)\ \mathrm{d}r \tag{4c}$$

$$L_3 = \int_e^a r\ \bar{k}_2 J_0\left(\frac{\omega r}{\alpha}\right) Y_0(\lambda_n e) J_0(\lambda_n r)\ \mathrm{d}r \tag{4d}$$

$$L_4 = \int_e^a r\ \bar{k}_2 J_0\left(\frac{\omega r}{\alpha}\right) J_0(\lambda_n e) Y_0(\lambda_n r)\ \mathrm{d}r \tag{4e}$$

$$\bar{k}_1 = \omega A\, J_0(\omega e/\alpha)/b_0 \tag{4f}$$

$$\bar{k}_2 = \omega A\, Y_0(\omega e/\alpha)/b_0 \tag{4g}$$

$$b_0 = J_0(\omega e/\alpha) Y_0(\omega a/\alpha) - Y_0(\omega e/\alpha)\ J_0(\omega a/\alpha) \tag{4h}$$

provided $\lambda_n \neq \omega/\alpha$. When $\lambda_n = \omega/\alpha = \beta$, we obtain[4]

$$A_\beta = \frac{\Pi^2 J_0(\beta e)[L_{1\beta} - L_{2\beta} - L_{3\beta} + L_{4\beta}]}{2\alpha[J_0^2(\beta e)/J_0^2(\beta_a) - 1]} \tag{5a}$$

where

$$L_{1\beta} = \bar{k}_1\ Y_0(\beta e) \int_e^a J_0(\beta r) Y_0(\beta r) r\ \mathrm{d}r \tag{5b}$$

$$L_{2\beta} = \bar{k}_1\ J_0(\beta e) \int_e^a Y_0(\beta r) Y_0(\beta r) r\ \mathrm{d}r \tag{5c}$$

$$L_{3\beta} = \bar{k}_2\ Y_0(\beta e) \int_e^a J_0(\beta r) J_0(\beta r) r\ \mathrm{d}r \tag{5d}$$

$$L_{4\beta} = \bar{k}_2\ Y_0(\beta e) \int_e^a Y_0(\beta r) J_0(\beta r) r\ \mathrm{d}r \tag{5e}$$

From Eqs. (3) and (4), we compute the values of λ_n and A_n for various values of n in Table 1. We note that the amplitudes A_n drop off sharply at high frequencies. The ratios of the frequencies are taken to cancel the effects of the scale factors (see Table 2).

Conclusions

The eigenfrequencies generated by the simple model compare favorably with the frequencies of the complex finite element model. Moreover, this analysis produces the amplitudes of the vibrating modes. These eigenfrequencies and eigenmodes could be used to study the dynamics and control system design of the hoop-column antenna as a flexible body.

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Errata

Finite Element Method for Optimal Guidance of an Advanced Launch Vehicle

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THE position descriptions for the authors of this paper were incorrectly identified. The AIAA Editorial Staff regrets this error and any inconvenience it has caused our readers. The correct information appears below:

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